Exponentials and logarithms 14E

1 a $\log_2(7\times 3) = \log_2 21$

b $\log_2\left(\frac{36}{4}\right) = \log_2 9$

c $3\log_5 2 = \log_5 2^3 = \log_5 8$

$\log_5 8 + \log_5 10 = \log_5(8\times 10) = \log_5 80$

d $2\log_5 8 = \log_6 8^2 = \log_6 64$

$4\log_6 3 = \log_6 3^4 = \log_6 81$

$\log_6 64 - \log_6 81 = \log_6\left(\frac{64}{81}\right)$

e $\log_{10} 5 + \log_{10} 6 = \log_{10}(5\times 6) = \log_{10} 30$

$\log_{10} 30 - \log_{10}\left(\frac{1}{4}\right) = \log_{10}\left(\frac{30}{\frac{1}{4}}\right)$

$= \log_{10} 120$

2 a $\log_2\left(\frac{40}{5}\right) = \log_2 8 = 3 \quad (2^3 = 8)$

b $\log_6(4\times 9) = \log_6 36 = 2 \quad (6^2 = 36)$

c $2\log_{12} 3 + 4\log_{12} 2$

$= \log_{12}(3^2) + \log_{12}(2^4)$

$= \log_{12} 9 + \log_{12} 16$

$= \log_{12}(9\times 16)$

$= \log_{12} 144$

$= 2 \quad (12^2 = 144)$

d $\log_8(25\times 10) - \log_8(5^3)$

$= \log_8 250 - \log_8 125$

$= \log_8\left(\frac{250}{125}\right)$

$= \log_8 2$

$= \frac{1}{3} \quad \left(8^{\frac{1}{3}} = 2\right)$

2 e $\log_{10}(2^2) - \log_{10}(5\times 8)$

$= \log_{10} 4 - \log_{10} 40$

$= \log_{10}\left(\frac{4}{40}\right)$

$= \log_{10}\left(\frac{1}{10}\right)$

$= -1 \quad \left(10^{-1} = \frac{1}{10}\right)$

3 a $\log_a x^3 + \log_a y^4 + \log_a z$

$= 3\log_a x + 4\log_a y + \log_a z$

b $\log_a x^5 - \log_a y^2$

$= 5\log_a x - 2\log_a y$

c $\log_a a^2 + \log_a x^2$

$= 2\log_a a + 2\log_a x$

$= 2 + 2\log_a x \quad (\log_a a = 1)$

d $\log_a\left(\frac{x}{\sqrt{yz}}\right)$

$= \log_a x - \log_a \sqrt{yz}$

$= \log_a x - (\log_a \sqrt{y} + \log_a z)$

$= \log_a x - \frac{1}{2}\log_a y - \log_a z$

e $\log_a\left((ax)^{\frac{1}{2}}\right)$

$= \frac{1}{2}\log_a(ax)$

$= \frac{1}{2}\log_a a + \frac{1}{2}\log_a x$

$= \frac{1}{2} + \frac{1}{2}\log_a x$

4 a $\log_2 3 + \log_2 x = 2$

$\log_2(3\times x) = 2$

$2^2 = 3x$

$x = \frac{2^2}{3}$

$= \frac{4}{3}$

4 b $\log_6 12 - \log_6 x = 3$

$\log_6\left(\frac{12}{x}\right) = 3$

$6^3 = \frac{12}{x}$

$x = \frac{12}{6^3} = \frac{1}{18}$

c $2\log_5 x = 1 + \log_5 6$

$2\log_5 x - \log_5 6 = 1$

$\log_5 x^2 - \log_5 6 = 1$

$\log_5\left(\frac{x^2}{6}\right) = 1$

$5^1 = \frac{x^2}{6}$

$x^2 = 30$

$x = \sqrt{30}$

d $2\log_9(x+1) = 2\log_9(2x-3) + 1$

$2\log_9(x+1) - 2\log_9(2x-3) = 1$

$\log_9(x+1)^2 - \log_9(2x-3)^2 = 1$

$\log_9\left(\frac{x+1}{2x-3}\right)^2 = 1$

$\left(\frac{x+1}{2x-3}\right)^2 = 9^1$

$\frac{x+1}{2x-3} = 3$

$x + 1 = 6x - 9$

$x = 2$

5 a $\log_3(x+1) = 1 + 2\log_3(x-1)$

$\log_3(x+1) - 2\log_3(x-1) = 1$

$\log_3(x+1) - \log_3(x-1)^2 = 1$

$\log_3 \frac{x+1}{(x-1)^2} = 1$

$\frac{x+1}{(x-1)^2} = 3^1$

$3(x-1)^2 = x + 1$

$3(x^2 - 2x + 1) = x + 1$

$3x^2 - 6x + 3 - x - 1 = 0$

$3x^2 - 7x + 2 = 0$

5 b $3x^2 - 7x + 2 = 0$

$(3x - 1)(x - 2) = 0$

$x = \frac{1}{3}$ or $x = 2$

The equation contains $\log(x - 1)$, so $x > 1$, therefore $x = 2$

6 $\log_6 a + \log_6 b = 2$

$\log_6(ab) = 2$

$6^2 = ab$

$ab = 36$

Rearrange $a + b = 13$

$13 - b = a$

Using substitution

$(13 - b)b = 36$

$13b - b^2 = 36$

$b^2 - 13b + 36 = 0$

$(b - 9)(b - 4) = 0$

$b = 9$ or 4

When $b = 9$, $a = 4$

When $b = 4$, $a = 9$

As $a > b$, $a = 9$ and $b = 4$

Challenge

$\log_a x = m$ and $\log_a y = n$

$a^m = x$ and $a^n = y$

$\frac{x}{y} = \frac{a^m}{a^n} = a^{m-n}$

$\log_a\left(\frac{x}{y}\right) = m - n = \log_a x - \log_a y$