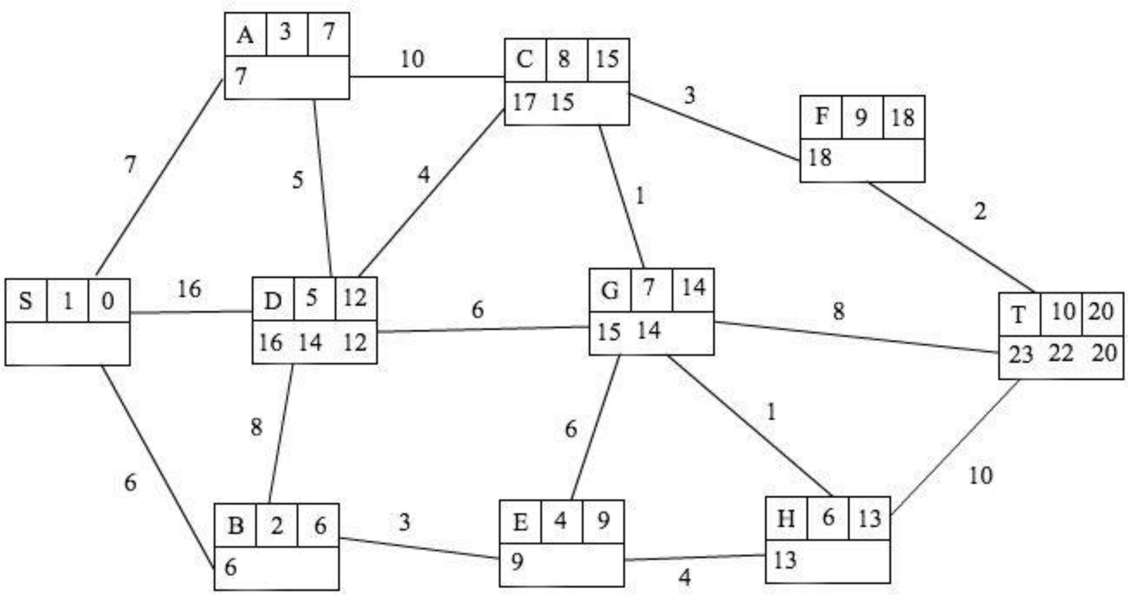
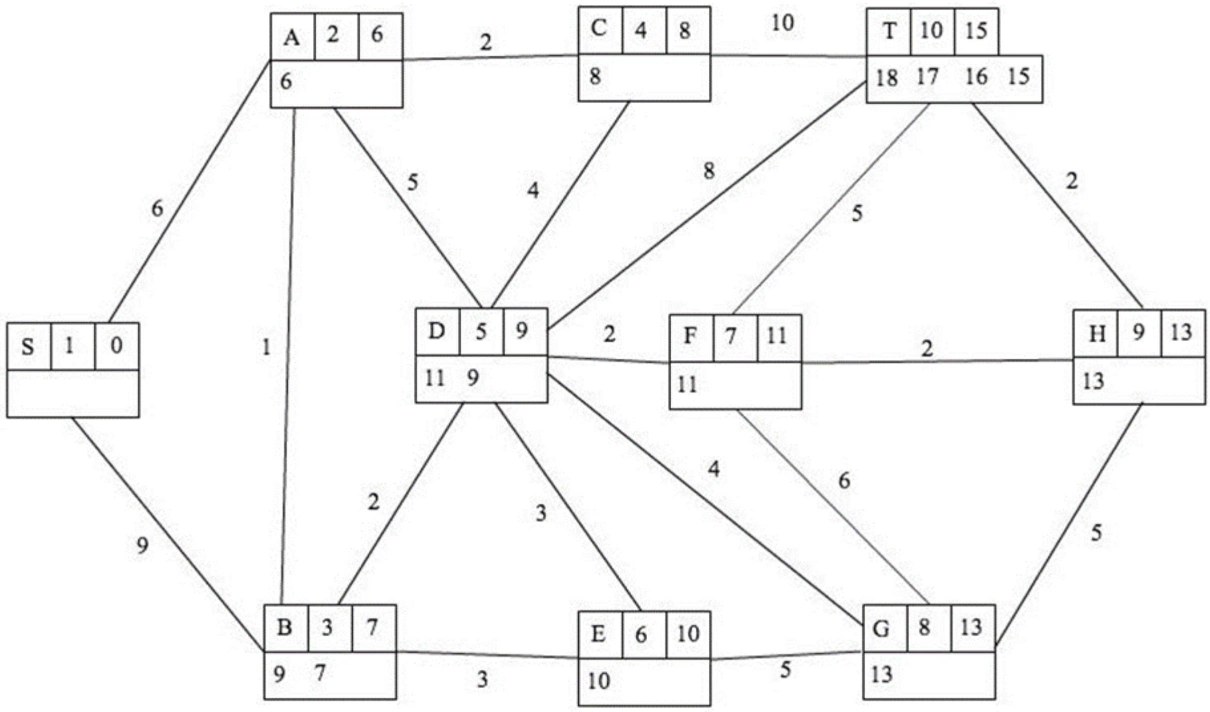
Algorithms on graphs 3D

1 a Use Dijkstra's algorithm to construct the following graph



So the shortest route from *S* to *T* has length 20. Now, to find this route, work backwards from *T*:
20 – 2 = 18 *TF*
18 – 3 = 15 *FC*
15 – 1 = 14 *CG*
14 – 1 = 13 *GH*
13 – 4 = 9 *HE*
9 – 3 = 6 *EB*
6 – 6 = 0 *BS*
Thus, the shortest route: *SBEHGCFT*. Length of shortest route: 20

1 b Use Dijkstra's algorithm to construct the following graph.

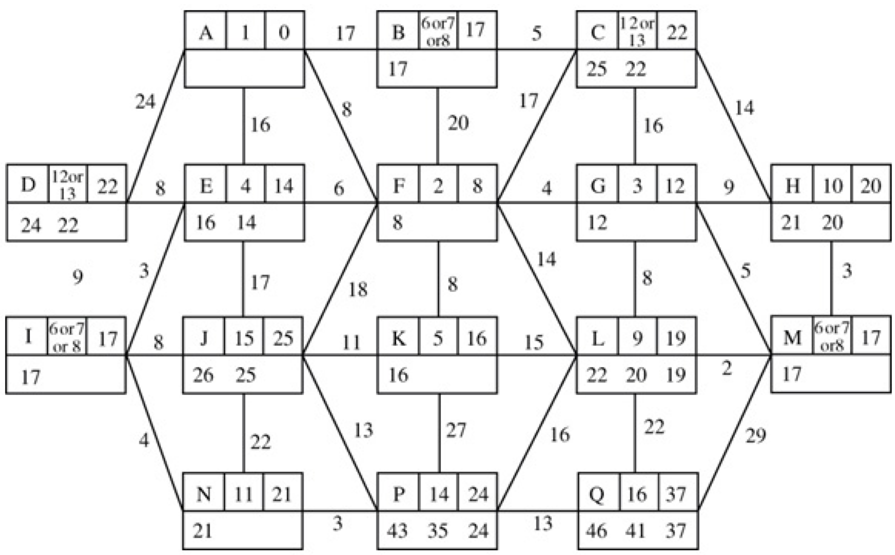


So the shortest route from S to T has length 15. To find the route, work backwards from T:

$15 - 2 = 13$ *TH*
$13 - 2 = 11$ *HF*
$11 - 2 = 9$ *FD*
$9 - 2 = 7$ *DB*
$9 - 9 = 0$ *BS*

Thus, the shortest route: *SABDFHT*. Length of shortest route: 15

2



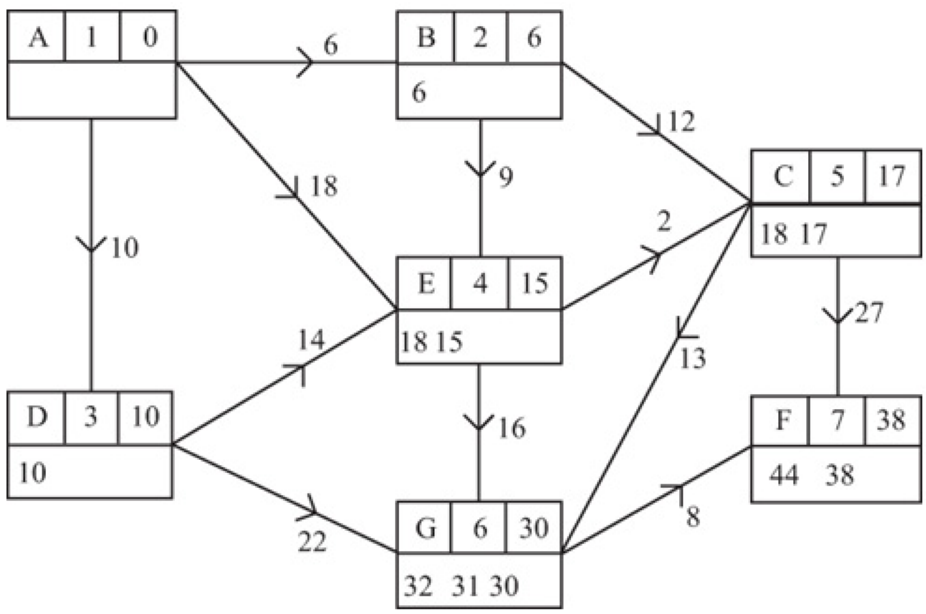
a A to Q   A – F – E – I – N – P – Q   Length   37

b A to L   A – F – G – M – L   Length   19

c M to A   M – G – F – A   Length   17

d P to A P – N – I – E – F – A Length 24

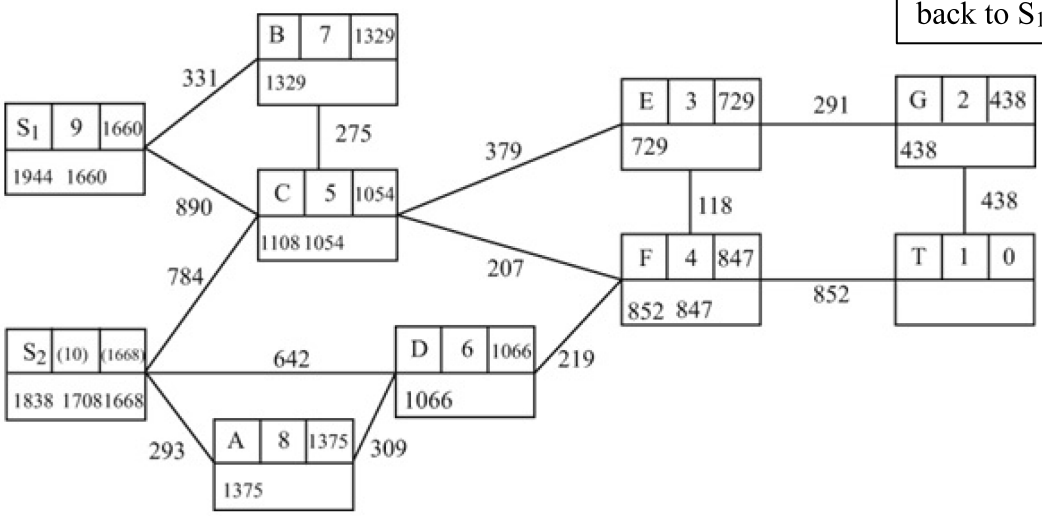
3



Shortest route: A – B – E – C – G – F Length 38

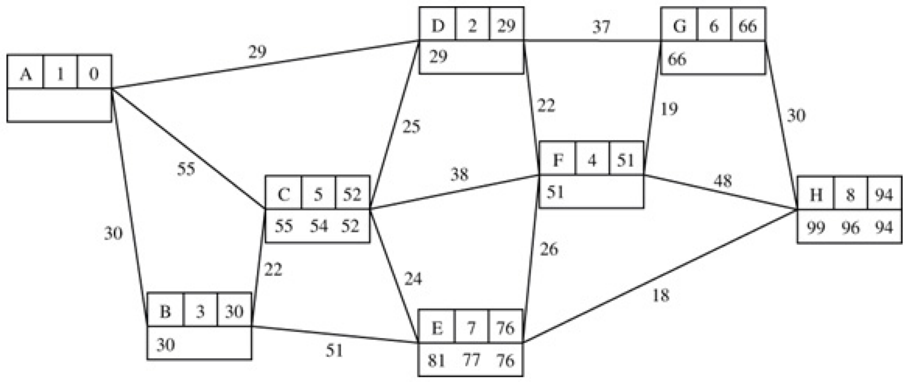
4

Start at T and work back to S_1 and S_2.



Shortest route: S_1 – B – C – F – E – G – T
Length of shortest route: 1660

5

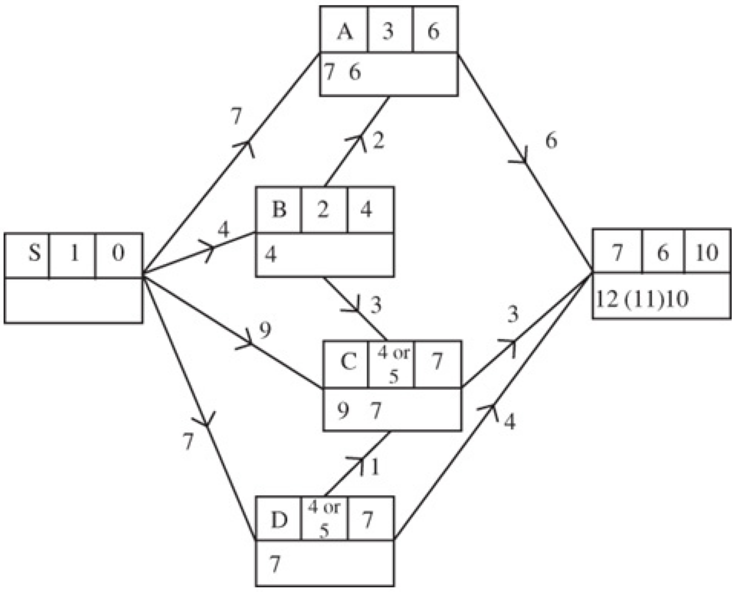


a $94 - 18 = 76$ EH
$76 - 24 = 52$ CE
$52 - 22 = 30$ BC
$30 - 30 = 0$ AB
Shortest route A to H: A – B – C – E – H Length 94

b Shortest route A to H via G: A – D – G – H Length 96

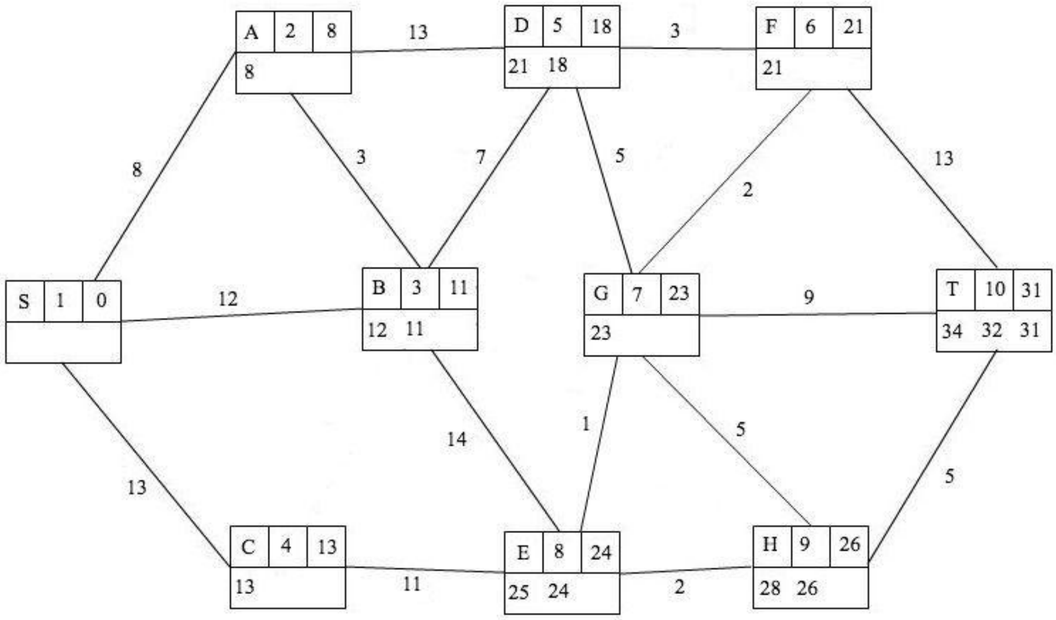
c Shortest route A to H not using CE: A – D – F – E – H Length 95

6



Shortest route: S – B – C – T
Length of shortest route: 10

7 a Use Dijkstra's algorithm to construct the following graph

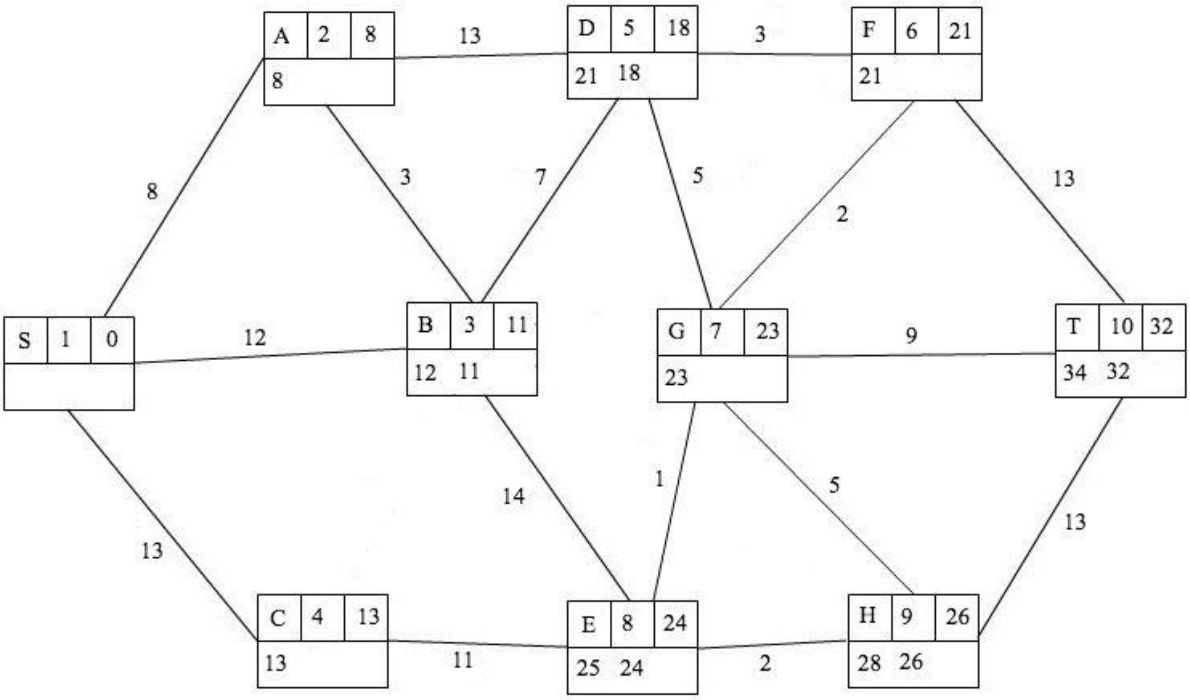


So the quickest route has length 31 minutes. To find the route, work backwards from T:

31 – 5 = 26	*TH*		
26 – 2 = 24	*HE*		
24 – 1 = 23	*EG*	or	24 – 11 = 13 *EC*
23 – 2 = 21	*GF*	or	13 – 13 = 0 *CS*
21 – 3 = 18	*FD*		
18 – 7 = 11	*DB*		
11 – 3 = 8	*BA*		
8 – 8 = 0*AS*			

So there are 2 routes of the same, shortest time: *SCEHT* and *SABDFGEHT*.
Shortest time = 31 min.

7 b i Use Dijkstra's algorithm to create the following graph



So the length of the journey changes to 32 minutes. To find the route, work backwards from *T*:

32 – 9 = 23 *TG*

23 – 2 = 21 *GF* or 23 – 5 = 18 *GD*

21 – 3 = 18 *FD* We reached point *D* so both routes coincide again

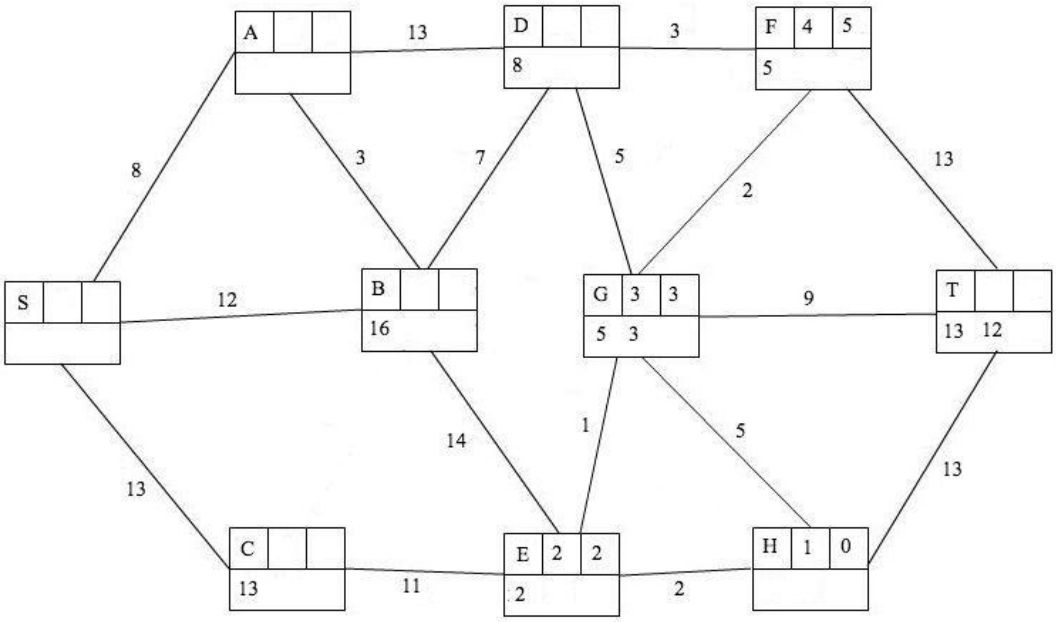
18 – 7 = 11 *DB*

11 – 3 = 8 *BA*

8 – 8 = 0 *AS*

So the route changes to *SABDFGT* or *SABDGT*, both of length 32 minutes.

7 b ii If the driver finds out about the change at *H,* that is his penultimate stop, we can consider the following graph



Now, even though the graph is unfinished (we have not considered what happens at vertices *A* through to *D*), we have exhausted all vertices directly connected to *T* and so any other route would eventually have to reach *F, G* or *H.* This means that any other route would necessarily be longer than what we can construct at the moment. Hence, the quickest route from *H* to *T* is 12 minutes. It can be found by working backwards from *T:*

$12 - 9 = 3$ *TG*

$3 - 1 = 2$ *GE*

$2 - 2 = 0$ *EH*

So instead of going from *H* to *T* directly, the driver should turn back and go *HEGT* to save 1 minute. This will make the total journey time 38 minutes (since from part **i** we know that the driver took 26 minutes to get to *H* and then from the graph above we have another 12 minutes to get from *H* to *T.*

c There are 10 different locations connecting the given network.

$$0.026 \times \left(\frac{40}{10}\right)^2 = 0.416 \text{ seconds}$$

d The time required is not directly proportional to the square of the number of locations, this is just an approximation.

